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## A SPATIAL ELECTION WITH COMMON VALUES \*

Carlos Maravall Rodríguez <sup>1</sup>

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### *Abstract*

Does electoral competition make candidates reveal information that voters value? I study this question in a Downsian model of a repeated election consistent with six stylized facts of US Presidential Elections: (i) there are two candidates/parties, (ii) they are longlived, (iii) there is majority rule, competition is over many issues at a time (iv) some on which voters disagree, (v) others on which they do not, and (vi) prior to the election, not all information that voters value is available to them. In this election, even if candidates compete in multidimensional space and appear ex-ante identical, Nash equilibria exist.

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**Keywords:** Spatial Elections, Imperfect Information, Common Values, Valence Issues

**JEL Classification:** C72, C73, D72, D82.

\* This paper is based on Chapter 3 of my Ph.D. dissertation at New York University. I am indebted to Jean-Pierre Benoît and Raquel Fernández for advice and encouragement with it. It was completed while I was a Pre-Dissertational Fellow at the Center for Business, Government, and Society at the Kellogg School of Management, Northwestern University. I thank Daniel Diermeier for inviting me and all members of the Center for their hospitality and support. Timothy Feddersen and seminar participants at NYU, Northwestern, the Midwest Political Science Association Conference in 2003 and 2004, and the 2004 Econometric Society European Meetings provided useful comments and suggestions. Financial support from the Fulbright Commission and Fundación ICO is gratefully acknowledged. Remaining errors are my own.

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# Introduction

Not all information that affects voters' preferences is necessarily available individually to each of them prior to an election. This is one reason why political participation is important: to aggregate the individual pieces of information held by voters. Even if all voters are not perfectly informed, the electoral outcome might contain the information they collectively hold. Condorcet (1785) is the first study of this question, but recent work by Feddersen and Pesendorfer (1996) has subtly refocused it. They show that, with two policy alternatives, absolute participation by all members in society is not necessarily relevant: less informed members can delegate their vote via abstention to more informed voters.

However, in actual elections voters rarely pick among policies, but among candidates who run to win an election. A less explored avenue of research is whether voters can trust candidates' ambition to win to become informed of all they need to know. In this approach what is critical is for candidates to be forced to reveal what they know to win the election. Voters are a passive third party and information is a side product that “trickles” down to them because of, in the subtle words of spanish folk wisdom, the competition “between hienas and jackals”.

In this paper I study if electoral competition makes candidates reveal information that voters value. Moreover, I allow candidates to offer platforms covering many issues at a time. To allow candidates to compete on many issues at a time when both are observed to be identical (apart from the platforms they offer) is normally problematic. For example, the standard Downsian model is generally prediction-free when platforms are defined in a multidimensional space, as it lacks Nash equilibria. In this paper I show that the presence of asymmetric in-

formation between candidates and voters determines equilibria can exist in a Downsian model where multiple issues are voted at once even if candidates are ex-ante identical.

The reason is that the election has an added value. Beyond the disagreement among voters in multidimensional space, it offers a way to pick the best among two candidates.<sup>1</sup> Even if voters cannot identify him prior to the election, they expect one candidate to be qualitatively better than the other on average, and voters get an added value from electing him that overcomes the level of disagreement among them in multidimensional space. For simplicity I assume that all voters share in common the value from the better candidate. However, the argument can be generalized: it need not be absolute, it is enough that a minority that is pivotal as in Feddersen and Pesendorfer (1996) shares the common value.

The model assumes six stylized facts characteristic of most US Presidential elections: (i) there are two parties/candidates, (ii) they are long-lived, (iii) the winner is elected by majority rule, there is unrestricted competition over many issues at a time, (iv) some among which voters disagree, (v) others among which they do not, and (vi) prior to the election, not all information that voters value is available to them. In particular, information is qualitative: it is relevant not for issues that are positional or relative, but for those in which voters can agree as better or worse about a candidate, i.e. a good record of economic performance, reducing crime or, instead, having been unable to deal with corrupt subordinates. That is, there is a qualitative dimension to candidates that is not observable prior to the election and the election is the mechanism through which it is revealed.

This is a Downsian model and as such it shares the central prediction of the classic one-dimensional model: a strong tendency for policy to converge to the median point (in this case a *generalized* median, as the policy space is multidimensional). However, there need not be full convergence: the expected difference in quality among candidates dictates how far can the winner's platform be from the median. Moreover, divergence is found in other models, but here it is based on Donald Stokes' claim that changes in policy can be unrelated to changes in the voting distribution (contrary to the classic Downsian model). Instead, and Stokes (1963) provides evidence to this effect, they can be determined by changes in the quality of elected officials.

Practical use of information is not something new in elections. Qualitative differences between candidates not observable prior to the election are common. An example is found in the summer of 1858, when Abraham Lincoln campaigned against Stephen Douglas in Illinois to be elected to the Senate. Some time that summer Lincoln must have realized that the election could be used to reveal information for the coming presidential election of 1860. Slavery at that time was already one of the most contentious issues in American politics. Lincoln's strategy was to force Douglas to answer if he would accept if citizens of new states excluded slavery from extending to their state. Douglas answered affirmatively and managed to retain his Illinois seat. He also managed to divide the Democratic party along Northern and Southern lines. The South nominated another Democratic candidate in 1860 and Lincoln won the presidency.

As Riker (1986) reveals, the qualitative (or valence) issue was not Lincoln's stand on slavery. Douglas also supported exclusion, and the country itself was divided. What is exemplar of Lincoln's political skill was his strategy to force Democratic candidates to reveal their individual

stand on slavery: the party at that time was not united. Lincoln managed to collect more votes than Douglas even if both stood for the same policy because Democratic partisanship faltered when forced to divide its vote.

## **Related Literature**

Recent literature (see Feddersen and Pesendorfer, 1996, and Razin, 2003) has analyzed elections where voters are not fully informed about some event that affects their preferences. In Condorcet (1785)'s vein, this literature sees elections as a mechanism to aggregate the bits and pieces of information dispersed among the population by voting. In this paper I instead study if electoral competition makes candidates reveal information that voters value. Previous literature (see Ledyard, 1989) has analyzed party competition to reveal information; but information that affects the preferences of candidates. Both here and in the literature on participation, information is modelled as a common value among voters (see Milgrom and Weber, 1982), i.e. a realization of a state of the world that affects the utility of all voters. Table 1 summarizes this first literature to which this paper belongs to.

The paper is also a contribution in the field of spatial elections defined in a multidimensional space (see Austen-Smith and Banks, 1999, and the references therein). The traditional way out of the existence problem is to argue that either there are institutional structures that determine that issues are only voted one at a time (see Shepsle, 1979), that voter actions are probabilistic (see Davis et al., 1970), that candidates cannot commit to any but their ideal point (see Besley and Coate, 1997), or that different platforms cannot be compared (see Roemer, 2001).

Instead, the seminal analysis of a multidimensional election with perfect information and a common value is Ansolabehere and Snyder (2000). They prove that an equilibrium exists if the common value (the “valence” or relative quality of a candidate) is larger than some minimal bound, the radius of the “yolk” (see McKelvey, 1986). Here I recast the multidimensional existence problem as a result of voter disagreement in multidimensional space with majority rule. The well-known Plott (1967) conditions then provide a measure of disagreement among voters’ ideal points. When they hold, there is zero disagreement among voters, and a Nash equilibrium exists whether or not there is a common value. In general, as in Ansolabehere and Snyder (2000), equilibria exist when there is a common value larger than the measure of disagreement among voters. The common value also parameterizes by how far can the winning candidate offer a divergent platform from the generalized median.

With asymmetric information, I assume that the election is repeated in each period with the same set of candidates/parties. I show that it has an outcome even if both candidates appear ex-ante identical to voters and compete in a multidimensional space. The reason is that electoral competition and the repetition of the election determine that, even if the high-quality candidate is not observable to voters, candidates are forced to reveal it. Thus, the model is an example of how to pick candidates in the presence of imperfect information (see Fearon, 1999). I construct a repeated election where prospective voting (picking the best of two observably identical candidates) and retrospective voting (past utility achieved with one of them) behavior interact. As the election is repeated, voters can put a candidate at a disadvantage if in the past he signaled being of higher quality and was not. This future disadvantage faced by the candidate is sufficient to ensure he never takes such action and, therefore, present signals are

fully informative.

Ferejohn (1986) is the first model with unobservable qualitative differences among candidates, as Rogoff (1990). Banks and Sundaram (1998) study a model with both adverse selection and moral hazard. What sets this model apart from theirs is competition on many issues at a time, and refocusing information revelation on *both* the incumbent and challenger. In the above literature, only the incumbent can signal his worth. In this model the challenger party is present across periods (as in U.S. Presidential elections) and this forces it whether or not to commit its efforts to win the current election. If it does not, it can take a chance and wait for the next election, but it always implicitly signals its choice.

The literature on observable qualitative differences among candidates has received new impetus with the theoretical work of Aragonès and Palfrey (2002) and Groseclose (2001), and the experimental work of Aragonès and Palfrey (2004). Their research builds on a long line of empirical work analyzing elections in the United States, e.g. Jacobson and Kernell (1981), Kiewiet (1983), and Krasno (1994). The seminal analysis on candidate qualitative differences of any kind is Stokes (1963).

The next section presents the model. Section three presents the equilibria when the high-quality candidate is publicly known, section four when it is only known by candidates.



# The Spatial Model

Consider an election with two candidates labelled  $P = \{D, R\}$ . Candidates can offer any platform in a multidimensional space  $\mathbf{X} \subseteq \Re^n$ , but must offer it with certainty, i.e. we only consider pure strategies. Candidates only care for winning, with benefit  $b$  if they do. When they tie each has an equal probability of winning.

Voters have to pick among the two candidates. Each voter  $i$  from the set  $I$  has an ideal point  $\mathbf{i} \in \mathbf{I}$ . The set of ideal points  $\mathbf{I} \subset \mathbf{X}$  is fixed and common knowledge, i.e. all candidates and voters know voters' ideal points. In the election there is a random variable  $S \subseteq \Re$  that determines voters value one candidate more than the other. All voters value this random variable in common and the distance of its realization,  $s$ , from 0 is the measure of value:  $R$  is “better” by  $s^2$  if  $s > 0$ ,  $D$  is “better” by  $s^2$  if  $s < 0$ . For simplicity, I assume that the conditional expectations of  $s$  on each side from 0 are equal:  $E[s|s > 0] = -E[s|s \leq 0] = \mu$ .

No candidate or voter observes the actual realization of  $s$ , but only a signal of it. With perfect information (section 3) I assume that candidates and voters observe the same signal  $\sigma$ , which is perfectly informative but qualitative. That is,  $\sigma$  tells without error whether  $s$  is positive or non-positive, but not by how much it differs from zero:

$$\Pr(\sigma_t > 0 | s_t > 0) = \Pr(\sigma_t \leq 0 | s_t \leq 0) = p = 1.$$

With imperfect information (section 4) candidates observe the signal  $\sigma$  of  $s$ , but voters do not observe anything. In this case, voters can only infer what candidates have learned by observing their actions.<sup>2</sup>

Voters' preferences are quadratic and additive, so if  $p$  is the candidate elected, they are

$$U(\mathbf{x}_p; (\mathbf{i}, s)) = -\|\mathbf{x}_p - \mathbf{i}\| + \begin{cases} s^2 & \text{if } s > 0 \text{ and } p = R, \text{ or} \\ & \text{if } s \leq 0 \text{ and } p = D. \\ 0 & \text{if } s \leq 0 \text{ and } p = R, \text{ or} \\ & \text{if } s > 0 \text{ and } p = D. \end{cases}$$

Thus, candidates and the bundle that consists of their platform and their relative quality according to  $s$ , are one and the same thing.

For voter  $i$ ,  $v_i \in [0, 1]$ , where  $v_i = 0$  indicates voting for candidate  $D$ ,  $v_i = 1$  for  $R$ , and  $v_i \in (0, 1)$  a mixed strategy. I also assume that for a particular voter, if he is indifferent between both candidates, he votes for the “better” one, in case he has some information on who that is.

With perfect information I consider pure strategy Nash equilibria but rule out those with weakly dominated strategies. With imperfect information I consider pure strategy perfect Bayesian Nash equilibria and also rule out those with weakly dominated strategies.

## Results with Perfect Information

In the election both candidates are free to make any offer in positional space but one of the candidates is endowed with a qualitative advantage that can be zero (if it is, in this sense, we are back to the standard Downsian case). When it is different from zero, one candidate differs from the other in the ability to provide utility to constituents. In political science this quality or ability is known as a “valence-issue” (see Stokes, 1963) and, in this section, I consider it is observable. The issue creates a wedge between platforms and the final utility voters obtain in the sense that voters obtain utility over and above the platform offered by the high-quality candidate just by electing him.

### The Game

First, nature picks a realization of  $s$  and the signal  $\sigma$  is observed by candidates and voters. Then each candidate is free to offer the bundle in  $\mathbf{X}$  he will implement if elected (commitment is credible, as candidates just want to win). Third, voters vote, and the winner is determined by majority rule and implements the policy he ran for. Finally, payoffs are realized.

### A Measure of Voter Disagreement in Multidimensional Space

It is useful to first analyse the case when voters do not expect any candidate to be any “better” than the other. That is, when they expect candidates to be qualitatively indifferent and the election has no Common Value ( $\mu = 0$ ). In this case, there is still the space  $\mathbf{X}$ , that has more than one dimension. This means that, in general, for any given voter distribution, it has more than one median hyperplane. With circular preferences, there is a one to one relation

between each majority and each median hyperplane, as each median hyperplane represents each possible majority that can be formed in this space. In this case, a well known condition for existence states that an equilibrium exists if and only if there is a point where all median hyperplanes intersect Davis et al. (1972). The reason is that, otherwise, for any point one considers, there is always a majority, defined by the median hyperplane to which this point does not belong, that prefers some other point. To state this condition it is necessary to introduce more notation.

Given the set of points  $\mathbf{I}$ , call  $\mathbf{M}(\mathbf{I})$  its set of median hyperplanes in  $\mathbf{X}$ . That is, for any  $\mathbf{x}' \in \mathbf{X}$  and  $a \in \mathbb{R}$ , define  $M(\mathbf{x}', a) = \{\mathbf{y} \in \mathbf{X} | \mathbf{y} \cdot \mathbf{x}' = a\}$ . So  $M(\mathbf{x}', a)$  is an  $n - 1$  dimensional hyperplane in  $\mathbf{X}$ . Then the hyperplane  $M(\mathbf{x}', a; \mathbf{I})$  is a median hyperplane if  $|\{\mathbf{i} \in \mathbf{I} | \mathbf{i} \cdot \mathbf{x}' < a\}| \leq \frac{|\mathbf{I}|}{2}$  and  $|\{\mathbf{i} \in \mathbf{I} | \mathbf{i} \cdot \mathbf{x}' > a\}| \geq \frac{|\mathbf{I}|}{2}$ . For simplicity I drop the indexation for a median hyperplane and just refer to  $M \in \mathbf{M}(\mathbf{I})$  as a median hyperplane.

For each point  $\mathbf{x} \in \mathbf{X}$  and each median hyperplane  $M$  in  $\mathbf{M}(\mathbf{I})$ , call  $d(\mathbf{x}, M)$  the distance between  $\mathbf{x}$  and  $M \in \mathbf{M}(\mathbf{I})$ , i.e.  $d(\mathbf{x}, M) = \min_{\mathbf{m} \in M \in \mathbf{M}(\mathbf{I})} \|\mathbf{x} - \mathbf{m}\|$ . Then define the maximum distance between  $\mathbf{x}$  and *any* median hyperplane as  $d_{\max}(\mathbf{x}; \mathbf{I}) = \max_{M \in \mathbf{M}(\mathbf{I})} d(\mathbf{x}, M)$ .

The first result in this paper restates the condition of Davis et al. (1972) in the following way: a point is an equilibrium if and only if its distance to any median hyperplane is zero.

**Theorem 1** *When  $\mu = 0$ ,  $(\mathbf{x}_D, \mathbf{x}_R)$  is a Nash equilibrium if and only if  $d_{\max}(\mathbf{x}_D = \mathbf{x}_R; \mathbf{I}) = 0$ .*

The theorem states that, in the absence of a common value, a policy is an equilibrium if and only if its measure of disagreement among voters is zero. Note that, in this equilibrium, both candidates must tie.

It follows that when the distribution of voters' ideal points satisfies the well-known Plott (1967) conditions, there is a point that belongs to all median hyperplanes, i.e.  $\min d_{\max}(\mathbf{x}; \mathbf{I}) = 0$ . Thus, it is important to know whether, for a given voter distribution, there is a point such that  $d_{\max}(\mathbf{x}; \mathbf{I}) = 0$ . The Plott distance gives the answer, as it tells exactly how distant is a given distribution from having an equilibrium when  $s = 0$ .

**Definition** Given a set of voter ideal points  $\mathbf{I}$ , its Plott distance is  $\min d_{\max}(\mathbf{x}; \mathbf{I})$ .

For simplicity, I drop the  $\mathbf{I}$  indexation from  $\mathbf{M}(\mathbf{I})$  and  $\min d_{\max}(\mathbf{x}; \mathbf{I})$ , so they become  $\mathbf{M}$  and  $\min d_{\max}(\mathbf{x})$ : it is understood that the set of median hyperplanes and the Plott distance are particular to each voter distribution (or set of ideal points  $\mathbf{I}$ ).

When there is no common value in the election, when no candidate is expected to be “better”, voters are personally indifferent among the two candidates. All they care for is the policy closest to their ideal point in  $\mathbf{X}$ . Instead, when voters expect the difference in quality between candidates,  $s$ , to be different from zero their behavior is altered. Now they face a tradeoff between picking the high-quality candidate and picking the candidate that offers a policy closest to their ideal point in  $\mathbf{X}$ . This tradeoff is what we analyze next. For the purpose of simplicity I will assume that  $s \leq 0$ . This analysis is without loss of generality. When  $s > 0$ , an analogous condition holds.

### **The Election with a Common Value, $\mu > 0$**

Assume voters observe  $\sigma \leq 0$  and they expect candidate  $D$  to be of higher-quality, as  $\mu > 0$ . Then, when a voter compares the offers between both candidates, she will bear in

mind that candidate  $D$  is the “better” candidate. This means that for every policy in space  $\mathbf{X}$  candidate  $D$  offers, it is as if he were  $\mu$  closer to every voter than the other candidate. That is, a particular voter will vote for  $D$  as long as the policy he offers,  $\mathbf{x}_D$ , is not further away from her ideal point than the policy candidate  $R$  offers,  $\mathbf{x}_R$ , plus  $\mu$ .

To find an equilibrium we need to consider the voting decisions of all voters. Remember that the median hyperplanes represent each majority we can consider in space  $\mathbf{X}$ . So, in particular, if the policy candidate  $D$  offers in  $\mathbf{X}$ ,  $\mathbf{x}_D$ , is not further away than  $\mu$  from *every* median hyperplane, he is a sure winner. In other words, candidate  $D$  is a sure winner if  $\mu - d_{\max}(\mathbf{x}_D) \geq 0$ .

The converse is also true: only policies in  $\mathbf{X}$  offered by candidate  $D$  not further away from some median hyperplane than the distance  $\mu$ , are an equilibrium. Otherwise, candidate  $R$  can offer a policy that wins. However, this policy is not an equilibrium either. The following result is proved in Ansolabehere and Snyder (2000) in a different setting. Though cast for circular preferences, an analogous result holds for any strictly quasi-concave preference (see Figure 1).

**Theorem 2** *When  $\mu > 0$ , and  $s \leq 0$*

$$(\mathbf{x}_D, \mathbf{x}_R) \text{ is a Nash Equilibrium if and only if } d_{\max}(\mathbf{x}_D) \leq \mu.$$

The theorem states that a policy is an equilibrium if and only if the common value is larger than the measure of disagreement in multidimensional space.

*Proof of Theorem 2.* (1) For a voter, electing candidate  $D$  is equivalent to electing a bundle  $\mathbf{x}$  that satisfies  $\mu - d_{\max}(\mathbf{x}) \geq 0$  and is expected to bring  $\mu$  extra utility. Instead, electing candidate  $R$  is equivalent to electing an unrestricted bundle  $\mathbf{x}$ , with no expectations about it. Consider the equation  $U(\mathbf{x}_D; (\mathbf{i}, s)) - U(\mathbf{x}_R; (\mathbf{i}, s)) = 0$ . It describes the set of voters indifferent between  $D$  and  $R$  and defines a hyperplane in  $\Re^n$  orthogonal to the vector  $\mathbf{x}_R - \mathbf{x}_D$ .

(2 - Sufficiency) Choose the origin and coordinate vectors in  $\Re^n$  so that  $\mathbf{x}_D = (0, 0, \dots, 0)$ ,  $\mathbf{x}_R = (x_{R1}, 0, \dots, 0)$ , and  $x_{R1} \geq 0$ . For any  $\mathbf{z} = (z_1, \dots, z_n) \in \mathbf{X}$ , define  $z_1(x_{R1})$  as the implicit function derived from  $[\mu^2 - \|\mathbf{x}_D - \mathbf{z}\|] - [-\|\mathbf{x}_R - \mathbf{z}\|] = 0$ . Note that  $z_1(x_{R1})$  is increasing, and  $\min z_1(x_{R1}) = \mu$ .

For a voter  $i$  with ideal point  $\mathbf{i} = (i_1, \dots, i_n)$ ,  $\mathbf{x}_D$  is preferred to  $\mathbf{x}_R$  if and only if  $i_1 \leq z_1(x_{R1})$ , otherwise he prefers  $\mathbf{x}_R$ . The same comparison can be made referring to the median voter defined by the hyperplane  $M_{yx} \in \mathbf{M}$  orthogonal to  $\mathbf{x}_R - \mathbf{x}_D$ , with intersection  $(m, 0, \dots, 0)$ . If this median voter prefers  $\mathbf{x}_D$ ,  $\mathbf{x}_D$  wins for sure, otherwise  $\mathbf{x}_R$  wins. To put it differently,  $\mathbf{x}_D$  wins with probability 1 if and only if  $m \leq z_1(x_{R1})$ . If  $d_{\max}(\mathbf{x}_D) \leq \mu$  this is true:

$$m = d(\mathbf{x}_D, M_{yx}) \leq d_{\max}(\mathbf{x}_D) \leq \mu \leq z_1(x_{R1}).$$

As  $\mathbf{x}_R$  is arbitrarily picked,  $\mathbf{x}_D$  wins against *any* policy offered by candidate  $R$ .

(3 - Necessity) Suppose not, suppose that  $d_{\max}(\mathbf{x}_D) > \mu$  and let  $M \in \mathbf{M}$  be the (or one of the) median hyperplane(s) such that this is true. Then there is a policy  $\mathbf{x}_R \in M$  preferred by a majority (the one defined by  $M$ ) to  $\mathbf{x}_D$ . ■

Note that the theorem only restricts the policy of the high-quality candidate and it states that he wins for sure. Moreover, what makes the inequality weak is the indifference-breaking rule.

I now consider the set of winning policies, which I call  $C$ , and compare it to the set of Pareto Optima, which I call  $PS$ .

## The Set of Winning Policies

I call  $C$  the set from which the “better” candidate can pick a bundle and win for sure when  $\sigma$  is observed by candidates and voters

**Definition**  $\mathbf{C} = \{\mathbf{x} \in \mathbf{X} \text{ s.t. } d_{\max}(\mathbf{x}) \in [\min d_{\max}(\mathbf{x}), \mu]\}$ , with boundary  $b_{\mathbf{C}}$ .

It is immediate to note that when  $\mathbf{C}$  is non-empty it is centered at  $\arg \min d_{\max}(\mathbf{x})$ . This is the sense in which the set of winning policies is centered at a generalized median.

**Definition** The generalized median is  $\arg \min d_{\max}(\mathbf{x})$ .

Note that the generalized median becomes the median when points are aligned (i.e. the one-dimensional case) and it always exists, even though it need not be a Nash equilibrium (when  $\mathbf{C}$  is empty).

Another property of  $\mathbf{C}$  is that it is convex. Moreover, the proof used below is a way to find and construct  $\mathbf{C}$  in general (and it is used for the examples later on).

**Proposition 1**  $\mathbf{C}$  is convex.

*Proof of Proposition 1.* For each median hyperplane construct a hyperplane parallel to it at a distance  $\mu$  on the side closest to  $\arg \min d_{\max}(\mathbf{x})$ . Consider the set that contains  $\arg \min d_{\max}(\mathbf{x})$  and all points bound by such parallel hyperplanes. This set is convex, as it is bound by hyperplanes. Moreover, no point inside this set does not belong to  $\mathbf{C}$ . It is precisely  $\mathbf{C}$ . ■



## Welfare

With Euclidean preferences it is well known that the set of Pareto optima is the convex hull of the ideal points.

**Proposition 2** *The set of Pareto optimal policies is the convex hull of the ideal points.*

$$PS = \left\{ \mathbf{x} \in \mathbf{X} \text{ s.t. } \mathbf{x} = \sum_{i \in I} \lambda_i \mathbf{i}_i, \sum_{i \in I} \lambda_i = 1, \text{ and } \lambda_i \in [0, 1] \ \forall i \right\}.$$

It should be intuitive that, when the set of possible equilibria,  $\mathbf{C}$ , is non-empty it always intersects the set of Pareto optima,  $PS$ , as the generalized median belongs to the convex hull of the ideal points. That is, when equilibria exist, some are Pareto optimal. However, when the set  $\mathbf{C}$  is sufficiently large, not all equilibria are Pareto optimal.

**Proposition 3** (i) *When  $\mu - d_{\max}(\mathbf{x}) \geq 0$ , some equilibria are Pareto optimal, and (ii) when  $\mu - d_{\max}(\mathbf{x}) \gg 0$ , not all equilibria are Pareto optimal.*

*Proof of Proposition 3.* (i) To prove that, if  $\mathbf{C} \neq \emptyset$ ,  $PS \cap \mathbf{C} \neq \emptyset$ , all we need is to prove that  $\arg \min d_{\max}(\mathbf{x}) \in PS$ . Consider only the set of median hyperplanes that intersect at least two ideal points:

i.i if there is only one such median hyperplane, then  $\arg \min d_{\max}(\mathbf{x})$  belongs to it and is in between at least two other ideal points.

i.ii if there is more than one such median hyperplane, then  $\arg \min d_{\max}(\mathbf{x})$  belongs inside the convex figure bound by this set of median hyperplanes.

(ii) For a given set of voter ideal points  $\mathbf{I}$ ,  $PS$  is fixed. Instead,  $\mathbf{C}$  increases with  $\mu$ . ■

The next subsection analyzes the bounds of the set  $\mathbf{C}$ .

## Bounds of the winning set

From Theorem 2 we know that an equilibrium exists if and only if  $\mathbf{C}$  is non-empty. The problem is that it is difficult to understand if this condition is restrictive or not, as  $\min d_{\max}(\mathbf{x})$  depends on the particular distribution of voters considered. That is, we do not know if it is very demanding to assume that the common value  $\mu$  is larger than  $\min d_{\max}(\mathbf{x})$ . Also, when  $\mathbf{C}$  is non-empty, we do not know how large it will be. In this subsection we relate the Plott distance to a geometric approach to find out how large is  $\min d_{\max}(\mathbf{x})$ .

Consider the ‘yolk’ (McKelvey, 1986). This is defined as the smallest closed ball in  $\mathbf{X}$  that intersects *all* median hyperplanes. Define  $r$ ,  $\mathbf{c}_y$ , and  $b_y$  as its radius, center, and boundary, respectively. The following proposition shows that the radius equals the Plott distance.

**Proposition 4** *(i)  $r = \min d_{\max}(\mathbf{x})$ , and (ii)  $\mathbf{c}_y = \arg \min d_{\max}(\mathbf{x})$ .*

*Proof of Proposition 4.* The radius of the smallest ball centered at any  $\mathbf{x} \in \mathbf{X}$  that intersects all median hyperplanes equals  $d_{\max}(\mathbf{x})$ . Such radius is greater or equal than  $r$ . ■

Let us now find how large is the Plott distance, or the radius of the yolk, in general. Feld et al. (1988) prove the following two results.

**Proposition 5** *The radius of the yolk is (i) at most one half of the radius of any ball which includes all voter ideal points, and (ii) is bounded above by the radius of a ball including a majority of the voter ideal points.*

The reader must judge for him- or herself if these conditions imply a large value of the Plott distance relative to the distances between voters’ ideal points.

To provide some intuition about the Plott distance, and the sets  $\mathbf{C}$ , and  $PS$  beyond these general results, the following two subsections consider some examples of voter distributions used in the voting literature.

## A One-dimensional Example

Figure 2 shows three voter ideal points,  $\mathbf{i}_1$ ,  $\mathbf{i}_2$ , and  $\mathbf{i}_3$ , aligned in space. Voter two is the median, and that would be the single Nash equilibrium in the absence of a common value, i.e. a high-quality candidate. In other words, it is immediate to show that  $\min d_{\max}(\mathbf{x}) = 0$  and  $\mathbf{i}_2 = \arg \min d_{\max}(\mathbf{x})$ . If we introduce a common value  $\mu$  as shown in the figure, the high-quality candidate can diverge from the median by as much as  $\mu$  and still be elected. Finally, the figure shows that the set of Pareto optima is the set of points in between all ideal points.

Note that only when voters preferences are circular and ideal points are aligned it is not binding to only consider policies on the line where ideal points lie (i.e. a one-dimensional space). Instead, it is binding if voter preferences are not circular. In this case, even if ideal points are aligned, the median is generally not a Nash equilibrium, unless policies are restricted to the one-dimensional space represented by the line where ideal points lie.<sup>3</sup>

## Two Multidimensional Examples

Figures 3 and 4 show two well-known examples of voter distributions defined on a plane. For instance, Figure 3 is found in chapter 5 of Austen-Smith and Banks (1999) and Figure 4 in McKelvey (1986). The figures show how large is  $\min d_{\max}(\mathbf{x})$  for these two examples, respectively. Figure 5 shows, at a smaller scale, how the Plott distance varies as the ideal points

of voters from Figure 3 become aligned. In the limit, they are identical to the distribution in Figure 2, and hence the Plott distance is zero.

Figures 6 and 7 compare the Plott distance to the distances between voters' ideal points. They emphasize the intuition born out of the Plott conditions: distances between ideal points are not relevant to determine  $\min d_{\max}(\mathbf{x})$ , but the distances between median hyperplanes are.

In figures 8 and 9 I assume a particular value of  $\mu$  such that, for these two voter distributions, I can draw  $\mathbf{C}$ , as it is non-empty. That is, I opt for a value of  $\mu$  such that  $\mu - \min d_{\max}(\mathbf{x}) \geq 0$  and a Nash Equilibrium exists in the election. In particular, in Figure 8 I first show the assumed value of  $\mu$  and then construct hyperplanes parallel to the median hyperplanes at precisely that distance. The set  $\mathbf{C}$  arises as the set bound precisely by these hyperplanes. From the figure it is obvious that only when  $\mu - \min d_{\max}(\mathbf{x})$  is greater or equal than zero is the set non-empty. The distribution of the second example defines many more median hyperplanes than the first. For this reason, in Figure 9 only the set of hyperplanes parallel to the median hyperplanes at the assumed distance  $\mu$  is shown. As in Figure 8, the set  $\mathbf{C}$  arises naturally as the set precisely bound by these hyperplanes. The generalized median,  $\arg \min d_{\max}(\mathbf{x})$ , is shown for both examples.

Finally, Figure 10 takes the first example and compares  $PS$  to the set  $\mathbf{C}$  depending on  $\mu$ .

## Results with Imperfect Information

With imperfect information the game is one in which one candidate has an expected qualitative advantage but it is not observable. To make information revealed I consider a repetition of the above game. I assume that over time the underlying quality of candidates changes randomly. Depending on economic, political, and social conditions, the high-quality recipe held by a party today can become its low-quality defining characteristic tomorrow, and viceversa. Candidates are identified with political parties and have preferences for winning intertemporally. The high-quality candidate has to signal his ability and in equilibrium it must be informative (i.e. the other candidate cannot signal that he is able too). That is, for the high-quality candidate to communicate his worth, the other candidate cannot profit from creating noise and distorting his signal. If candidates/parties have a high preference for winning across time rather than the one-shot present election, the present signal will be informative. The low-quality party of today realises that in the future he might be the high-quality alternative, and will not want to risk distorting the mechanism through which information is revealed today.

### The Game

At each period  $t$  an election is held. Within it, first nature picks a realization of  $s_t$  and candidates observe  $\sigma_t$  and voters do not. Each candidate then offers the platform he will implement if elected. Third, voters try to infer the sign of  $s$  from candidates' actions, vote, and the winner is elected by majority rule implementing the policy he ran for. Finally, payoffs are realized and voters infer the quality of the winner by calculating the difference between their payoff and the payoff from the winning platform itself.

I examine the equilibria of the game where the above election is repeated in each period. Candidates and voters discount the future at a common discount rate  $\delta$ , the realization of  $s$  in each period is independent of all others, and it is equally likely that  $s$  is positive or non-positive:

$$\Pr(s_t | s_{t-1}) = \Pr(s_t), \text{ and } \Pr(s_t > 0) = \Pr(s_t \leq 0).$$

This stochastic process ensures the election is well-defined as a repeated game: the payoff structure of the game is unaltered by agents' actions.

## A Fully Revealing Perfect Bayesian Equilibrium

### Strategies and beliefs

Let  $\mathbf{x}_{jt} : S \times \mathbf{H} \rightarrow \mathbf{X}$  be the pure strategy of candidate  $j = D, R$  upon observing state  $s_t$ , given a history  $h_t$ . Let  $\varphi_t : (\mathbf{X} \times \mathbf{X}) \times \mathbf{H} \rightarrow [0, 1]$  be the belief function of voters, i.e. the probability they believe candidate  $R$  is high-quality after observing candidates' offers  $(\mathbf{x}_{Dt}, \mathbf{x}_{Rt})$ , given a history  $h_t$ . Finally, the strategy of a voter is a mapping  $v_{it} : (\mathbf{X} \times \mathbf{X}) \times \mathbf{H} \rightarrow [0, 1]$ .

Strictly speaking, as a voter does not observe the signal  $\sigma$ , she does not have a preferred bundle, but a preferred bundle conditional on her belief of the sign of  $s$ . To keep things simple, I will avoid using the conditional expectation notation, and just name her conditional preferred bundle  $(\mathbf{i}, \varphi)$ , and  $(\mathbf{x}_p, p)$  the bundle implemented.

### The Equilibrium

**Theorem 3** *There is a  $\delta$  and an assessment  $\{\mathbf{x}_{Dt}^*, \mathbf{x}_{Rt}^*, \varphi_t^*, \{v_{it}^*\}_{t=0}^\infty\}$  such that, in equilibrium,*

- (i) the high-quality candidate is elected as (ii) he implements a policy in  $\mathbf{C}$ , and (iii) voters infer the sign of  $s$ .*

The Fully Revealing Perfect Bayesian Equilibrium described above is constructed assuming a discount rate of voters and candidates  $\delta$  larger than some minimum  $\delta^*$  and an assessment that ensures that the high-quality candidate offers a particular policy in  $\mathbf{C}$  and his behavior is not imitated. That is, *in each period  $t$ , the high-quality candidate offers a particular point  $\mathbf{x} \in \mathbf{C}$ , the other candidate offers a different point, and the high-quality candidate is elected.*

### Intuition

With imperfect information this is a signaling game between the candidates and the electorate and platforms are not just offers, but also signals to voters. Thus, for a candidate to signal his worth, a discontinuity on the set of beliefs must be created endogenously depending on the platforms offered. However, if the Plott distance is not zero, this is not trivial. If voters hold a belief equal to the prior after some deviation in a subgame, we are not sure an equilibrium exists in the subgame, precisely because the Plott distance is positive, and both candidates appear identical.

To make information revealed I assign two sets of points to each candidate. These sets are constructed so that if one of them offers a platform belonging to the set he is assigned, it is seen as a signal that he is the high-quality candidate. Both sets of points are different, so candidates cannot be confused by sitting on top of each other: if a candidate does not offer a platform from his assigned set he is not signaling he is the high-quality type. Then, I pick the two sets of platforms such that all points in one of the sets beat pairwise those in the other, in case voters hold a belief equal to the prior. Thus, if both candidates try to signal they are the high-quality one, one always wins.

Finally, from the two sets of points above, voters can potentially keep on assigning the

“bad” set of points to a candidate across periods, if in the previous period he decided to signal he was high-quality when he was not. If they do this, then the other always has an incentive to signal he is the high-quality candidate. With this assignment of sets voters can enforce past deviations from the fully revealing equilibrium path. Then no candidate has an incentive to deviate and signal he is high when he is not in the first place and the above assignment that puts a candidate at a disadvantage is just an out of equilibrium observation.

Figure 11 shows two examples of the above sets of points for each candidate when the distribution of voter ideal points is as in Figure 3. It is obvious to notice that one can always find such pairs of sets, as long as the set  $\mathbf{C}$  has a positive measure, i.e. as long as  $\mu > \min d_{\max}(\mathbf{x})$ . Figure 12 shows what is observed in the equilibrium path of the game. Finally, Figure 13 shows how the one-dimensional case would look.

*Proof of Theorem 3. 1. Argument*

In each period  $t$ , pick two sets of points in  $\mathbf{C}$ ,  $\mathbf{Z}_t$  and  $\mathbf{Z}'_t$  with  $\mathbf{Z}_t \neq \mathbf{Z}'_t$ , and assume that if the same candidate were to offer points from both, all points in  $\mathbf{Z}_t$  would be preferred by a majority to all those in  $\mathbf{Z}'_t$ :

*I.- Assign each from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$  to the candidates with the following rule: (i) if in the last election the high-quality candidate was elected, assign  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$  arbitrarily to any candidate; (ii) if in the last election the high-quality candidate was not elected, in the next  $N^*$  periods assign  $\mathbf{Z}_t$  to the high-quality candidate and  $\mathbf{Z}'_t$  to the winner in that election.*

*II.- Consider the following set of beliefs for a voter: (i) if no candidate sits at any point in his assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$  my belief is equal to my prior. That is, I believe both candidates are equally likely to be high-quality. (ii) If one candidate sits at any point in his assigned set from*



$\{\mathbf{Z}_t, \mathbf{Z}'_t\}$  and the other sits anywhere else but his, I believe that the high-quality candidate is him sitting at his assigned set. (iii) If both candidates sit at any point in their assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$ , my belief equals the prior.

III.- *The strategy of a candidate when he observes the realization of the state is:* offer any policy in my assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$  when I am the high-quality candidate, offer any other when I am not.

IV.- *Finally, the strategy of a generic voter  $i \in I$  is:* vote for him who gives me highest utility, conditional on whom I believe is the high-quality candidate.

## 2. Discussion

In general, in this multidimensional policy space, when voters hold a belief equal to their prior, we are not assured an equilibrium exists. However, the argument above ensures that the high-quality candidate always wants to sit at a point in his assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$ . Thus, voters would only hold their prior belief if the candidate inconsistent with the observed signal  $\sigma$  imitated the high-quality candidate and also sat at a point in his assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$ . In this case, an equilibrium exists: by assumption, all points in  $\mathbf{Z}_t$  beat those in  $\mathbf{Z}'_t$ . Thus, the payoff of this subgame is well-defined. However, as voters assign the set  $\mathbf{Z}'_t$  for  $N^*$  periods thereafter to this candidate (who imitates the high-quality candidate in that period), they are assured that he will never want to take this path and imitate the high-quality candidate in the first place. His action would be remembered for so many periods, benefitting the other candidate at his expense, that he would not want to take it. This is constructed with the following condition on  $\delta^*$  and  $N^*$ .

3. Define  $\delta^*$  and  $N^*$  such that

$$\frac{1}{2} \frac{\delta^*}{1 - \delta^*} b \geq b + \frac{1}{2} \frac{\delta^*}{1 - \delta^*} (\delta^*)^{N^*} b,$$

notice that  $\delta^* \in [\frac{2}{3}, 1]$ . Given the assignment  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$ , this expression is the tradeoff the candidate who is not the high-quality one faces. The left hand side of the expression is the expected discounted payoff if he *does not create noise*, if he does not sit today at a point in his assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$ . This is the fully revealing equilibrium path outlined in the argument above. In all future elections this candidate expects to win half of the time. Instead, the right hand side is the payoff he gets today if he *creates noise*, if he sits at a point in his assigned set from  $\{\mathbf{Z}_t, \mathbf{Z}'_t\}$ . In this case he distorts the mechanism through which information is revealed. In the present this candidate can at most win, but in the future he will lose for the next  $N^*$  periods and then return to the fully revealing path thereafter.

#### 4. Formal Construction

We now construct a Perfect Bayesian Equilibrium assessment based on the argument above with an assumed sufficiently large discount value.

Assume  $\delta \geq \delta^*$  and the following assessment  $\{\mathbf{x}_{Dt}^*, \mathbf{x}_{Rt}^*, \varphi_t^*, \{v_{it}^*\}\}$  :

I.- Location of candidates that determine beliefs ( $p_{t-1}$  is the previous period winner):

$$\begin{aligned} & \text{if } \left\{ \begin{array}{l} \sigma_{t-1} > 0 \text{ and } p_{t-1} = R, \text{ or} \\ \sigma_{t-1} \leq 0 \text{ and } p_{t-1} = D, \end{array} \right\} \text{ then let } \{\mathbf{x}_{Dt}^C, \mathbf{x}_{Rt}^C\} \in \{\mathbf{Z}_t, \mathbf{Z}'_t\}, \text{ with } \mathbf{x}_{Dt}^C \neq \mathbf{x}_{Rt}^C; \\ & \text{if } \left\{ \begin{array}{l} \sigma_{t-1} \leq 0 \text{ and } p_{t-1} = R, \text{ or} \\ \sigma_{t-1} > 0 \text{ and } p_{t-1} = D, \end{array} \right\} \text{ then let } \left\{ \begin{array}{l} \mathbf{x}_{Dt+k}^C \in \mathbf{Z}_t \\ \mathbf{x}_{Rt+k}^C \in \mathbf{Z}'_t \end{array} \right. \text{ if } \sigma_{t-1} \leq 0 \\ \left. \begin{array}{l} \mathbf{x}_{Rt+k}^C \in \mathbf{Z}_t \\ \mathbf{x}_{Dt+k}^C \in \mathbf{Z}'_t \end{array} \right. \text{ if } \sigma_{t-1} > 0 \end{array} \right\}, \text{ for } k = 0, \dots, N^*, \end{aligned}$$

II.- Beliefs depending on the location of the candidates.

$$\varphi_t^* = \begin{cases} 1 & \text{if } \mathbf{x}_{Rt} = \mathbf{x}_{Rt}^C, \mathbf{x}_{Dt} \neq \mathbf{x}_{Dt}^C, \\ 0 & \text{if } \mathbf{x}_{Dt} = \mathbf{x}_{Dt}^C, \mathbf{x}_{Rt} \neq \mathbf{x}_{Rt}^C, \\ \frac{1}{2} & \text{otherwise,} \end{cases}$$

III.- Strategies of candidates.

$$\mathbf{x}_{Dt}^* \in \begin{cases} \{\mathbf{x}_{Dt}^C\} & \text{if } \sigma_t \leq 0 \\ \mathbf{x} \in \mathbf{X} / \{\mathbf{x}_{Dt}^C\} & \text{if } \sigma_t > 0 \end{cases}, \mathbf{x}_{Rt}^* = \begin{cases} \{\mathbf{x}_{Rt}^C\} & \text{if } \sigma_t > 0 \\ \mathbf{x} \in \mathbf{X} / \{\mathbf{x}_{Rt}^C\} & \text{if } \sigma_t \leq 0 \end{cases},$$

IV.- Strategy of a generic voter.

$$v_{it}^* = \begin{cases} R & \text{if } \begin{cases} U[(\mathbf{x}_{Rt}, R); (\mathbf{i}, \varphi_t^*)] > U[(\mathbf{x}_{Dt}, D); (\mathbf{i}, \varphi_t^*)], \text{ or} \\ U[(\mathbf{x}_{Rt}, R); (\mathbf{i}, \varphi_t^*)] = U[(\mathbf{x}_{Dt}, D); (\mathbf{i}, \varphi_t^*)] \text{ and } \varphi_t^* > \frac{1}{2}, \end{cases} \\ D & \text{if } \begin{cases} U[(\mathbf{x}_{Rt}, R); (\mathbf{i}, \varphi_t^*)] < U[(\mathbf{x}_{Dt}, D); (\mathbf{i}, \varphi_t^*)], \text{ or} \\ U[(\mathbf{x}_{Rt}, R); (\mathbf{i}, \varphi_t^*)] = U[(\mathbf{x}_{Dt}, D); (\mathbf{i}, \varphi_t^*)] \text{ and } \varphi_t^* < \frac{1}{2}, \end{cases} \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

■

## Full Information Equivalence

In the proof of Theorem 3 we picked two arbitrary sets of points in  $\mathbf{C}$ ,  $\mathbf{Z}_t$  and  $\mathbf{Z}'_t$ , and all we asked them to satisfy is to be different, and all points in one to beat the other's. Thus, it follows that all equilibria with perfect information can be generated with imperfect information.

**Proposition 6** *Equilibrium policies implemented with perfect information can be implemented with imperfect information.*

*Proof of Proposition 6.* All we require from  $\mathbf{Z}_t$ ,  $\mathbf{Z}'_t$  is that one beat the other's points when voters' beliefs equal the prior. ■

## Empirical Implications

This is a Downsian model, so a very strong tendency to converge to the median, the center of the set  $\mathbf{C}$ , lies in it. This is the case whether an equilibrium exists or not (i.e. whether  $\mathbf{C}$  is empty or not, as shown in McKelvey (1986)). However, the differences in quality between candidates allow the high quality candidate to pick a policy that diverges from the median, as long as it is not further away from it than his qualitative advantage minus the level of disagreement among voters. Thus, whilst there are multiple equilibria and, thus, divergence from the median, they are bounded precisely by the difference in the qualitative characteristic of candidates.

The existence of multiple equilibria in this model should be seen as a plus: changes in policy when a new candidate enters office can be unrelated to changes in the voting distribution, but due to differences in the quality of the exiting and entering candidates.

## Discussion

This repeated election is an example of a mechanism voters can use to pick a candidate in the presence of imperfect information (see Fearon, 1999). In this case, the mechanism is one in which the disadvantage the “worse” candidate faces if he deviates out of the equilibrium path, ensures a smooth selection of the “better” candidate in equilibrium in each period. That is, the retrospective punishing behavior of voters in case candidates deviate ensures a smooth prospective voting behavior in equilibrium.

The election has a number of features that make it attractive. First of all, it is a model that can explain electoral competition in a multidimensional space. Second, it predicts platform divergence along the lines defended in Stokes (1963): differences in candidate quality allow for divergence without any change in the voting distribution. Moreover, the election has a clear value added: it is a mechanism to reveal the “better” candidate and elect him if he is moderate enough relative to his worth. This is because party competition in a repeated election context is sufficiently powerful to make all information revealed. Finally, it also shows that a candidate who is much preferred to another can win implementing policies that are not Pareto optimal: even if all information is revealed, outcomes are not necessarily efficient.

In its current state the model has two disadvantages, however. It predicts that in all equilibria the “better” candidate will win with probability one. Moreover, the position of the loser remains indeterminate, as he just does not want to be seen as the “better” candidate. For this reason, I plan to extend the model to have imperfect information in the sense mentioned in the paper. Voters, instead of being endowed with a perfect signal at the end of every election, will get an imperfect signal, i.e. one that is informative but not with unit probability. It is

expected that this weakening of the information structure in the model will allow to get rid of the two disadvantages mentioned above.

## Notes

1. Candidates differ in quality just as workers differ in productivity.
2. A note on imperfectly informative signals ( $p < 1$ ): when information revelation is on the side of candidates, the purpose of analyzing them is not evident. In any election the number of candidates is always relatively small, so the amount of information they hold collectively always remains imperfectly informative.
3. I thank David Austen-Smith for making this point.

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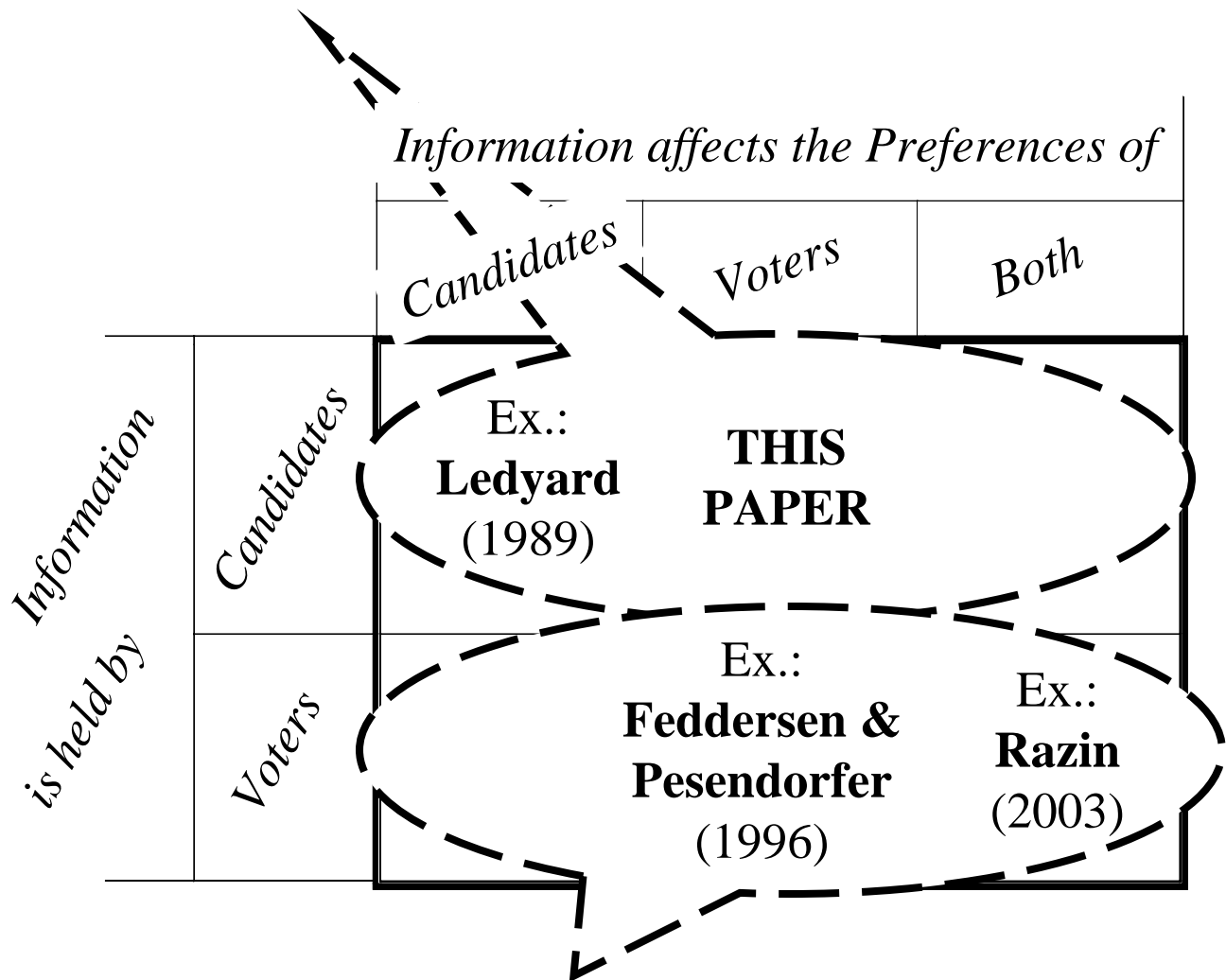
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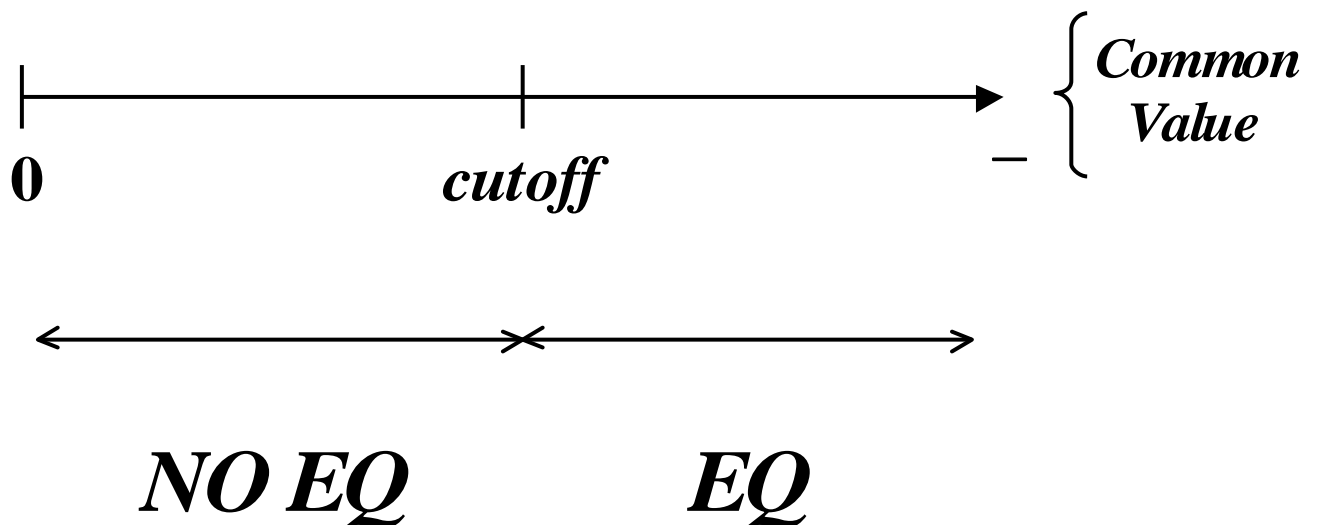
## Do elections make candidates reveal information?



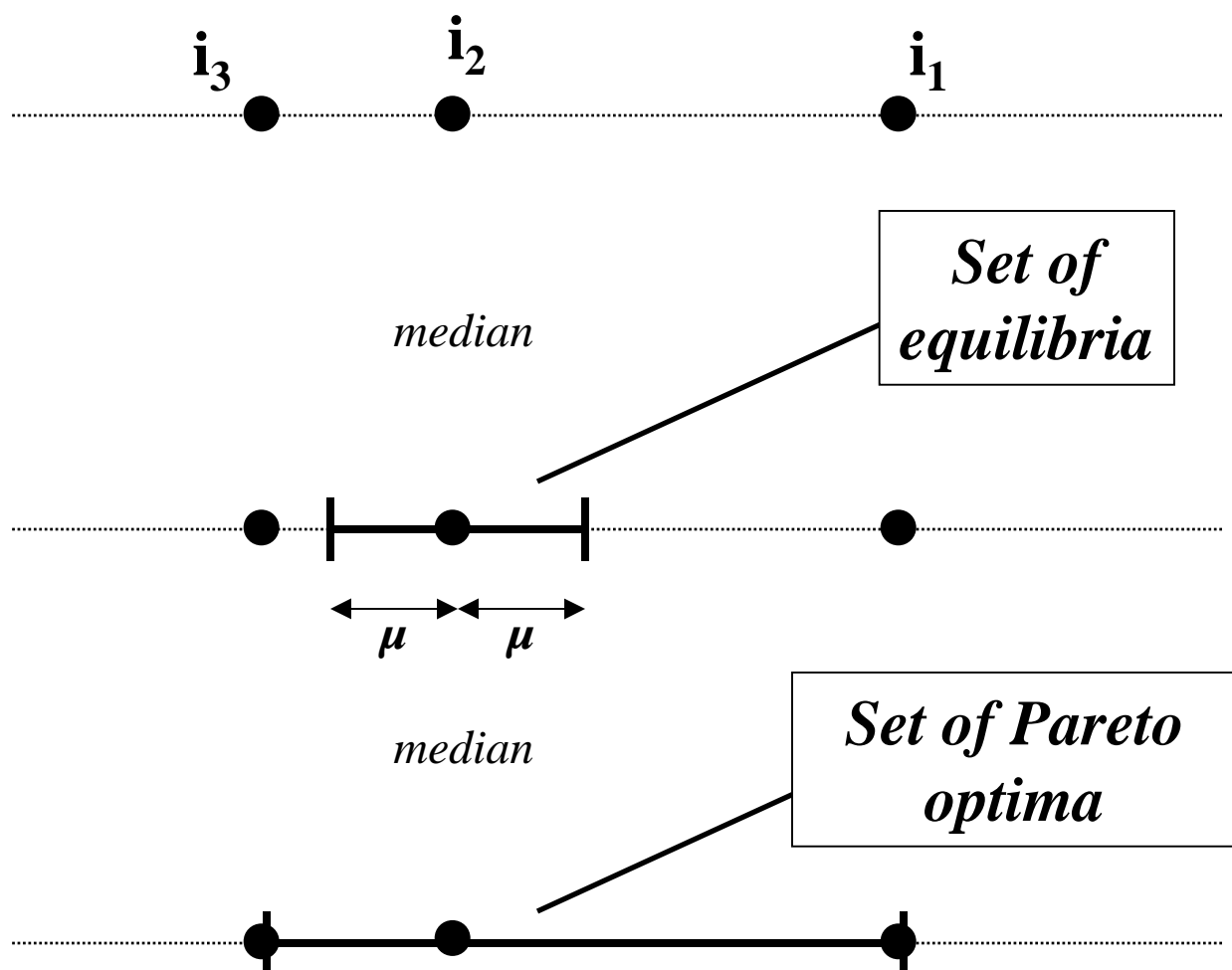
## Can elections aggregate information held by voters ?

**Table 1.** Literature on imperfect information

*The following holds irrespective of preferences being Euclidean:  
for the cutoff to exist, the sets  
centered at voters' ideal points  
and bounded by their indifference  
curves only need to be compact.*

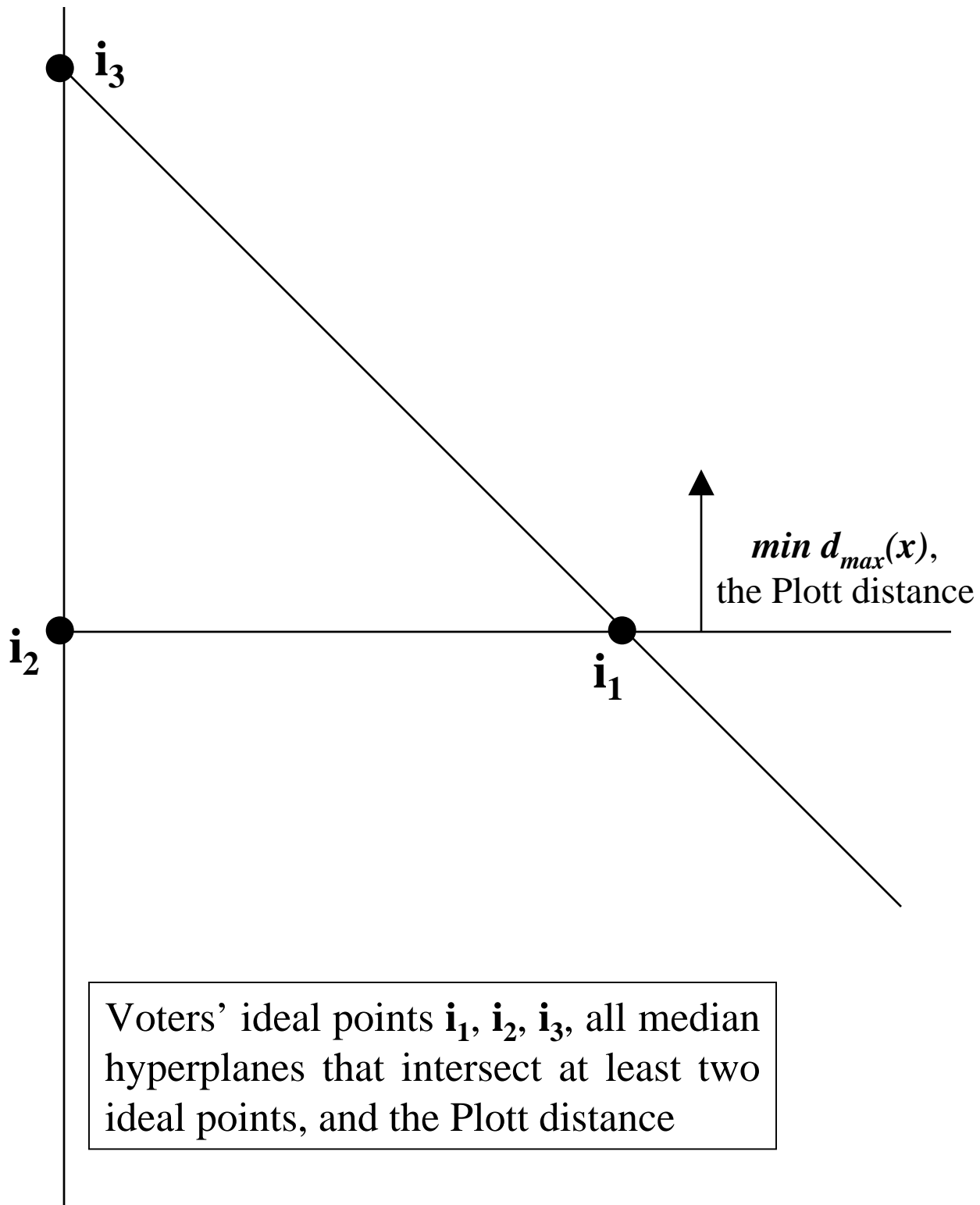


**Figure 1.** Being general



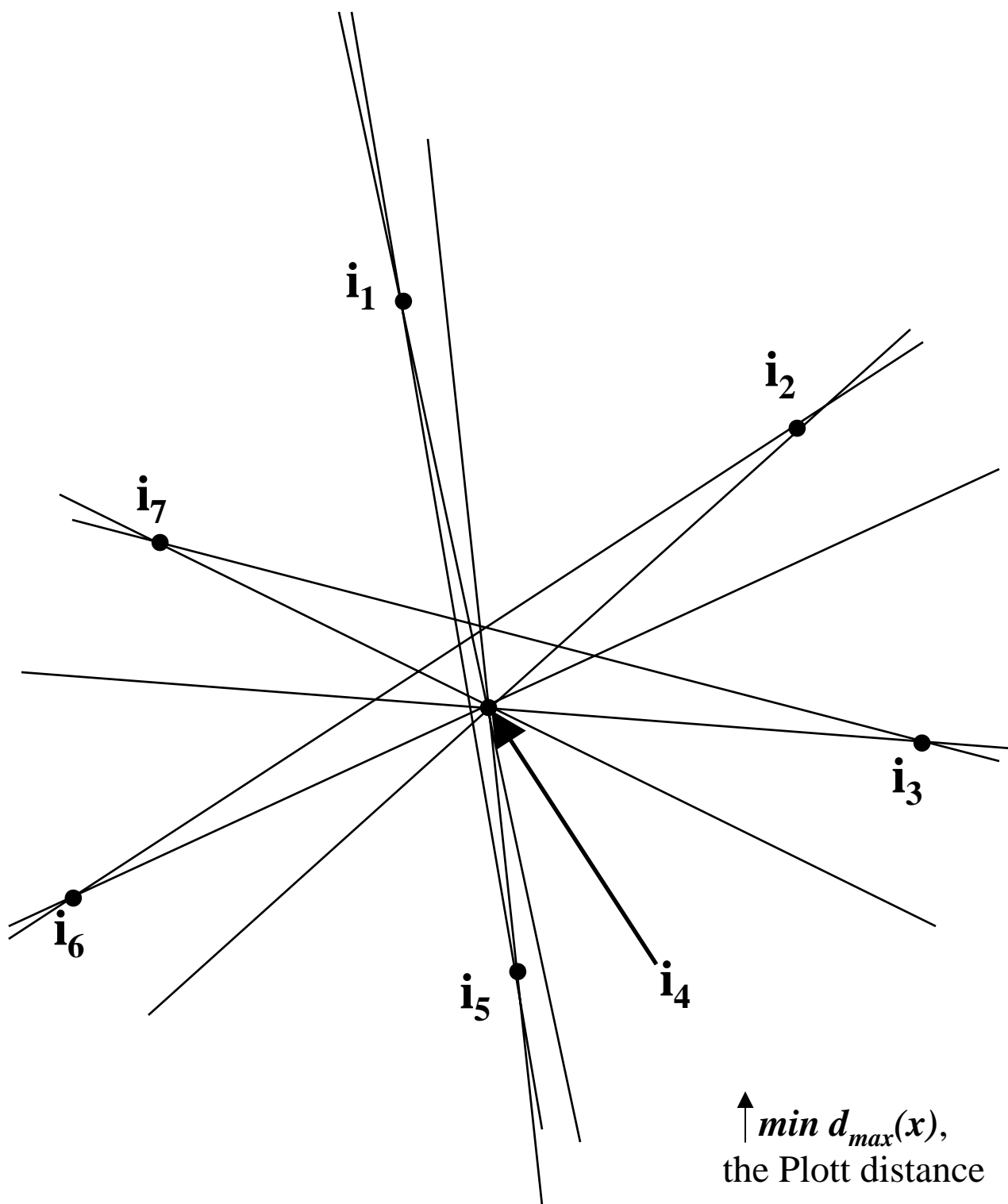
Voters' ideal points  $i_1$ ,  $i_2$ ,  $i_3$ , are aligned, so the Plott distance ( $\min d_{\max}(x)$ ) is 0, and the convex hull is just the convex interval containing all ideal points.

**Figure 2.** A one dimensional example



**Figure 3.** Multidimensional example 1

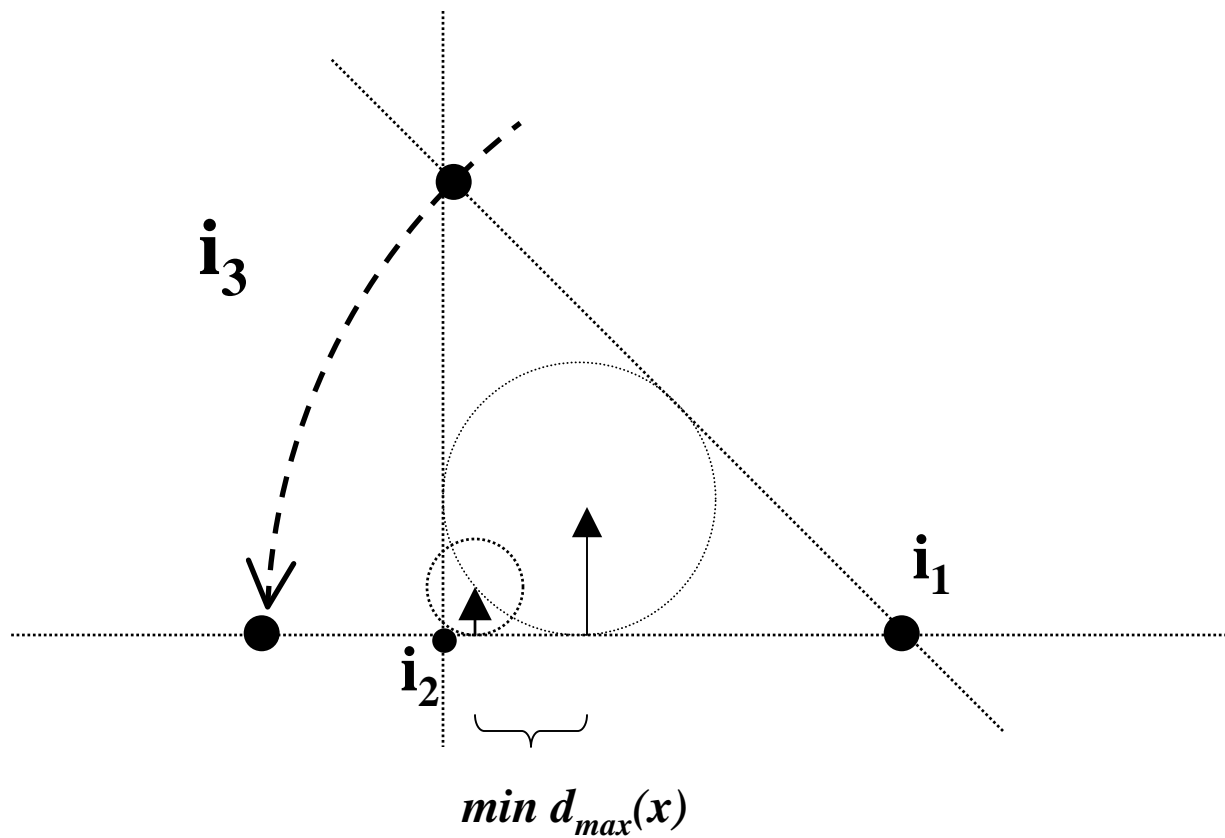
Voters' ideal points  $\mathbf{i}_1, \dots, \mathbf{i}_7$ , all median hyperplanes that intersect at least two ideal points, and the Plott distance.



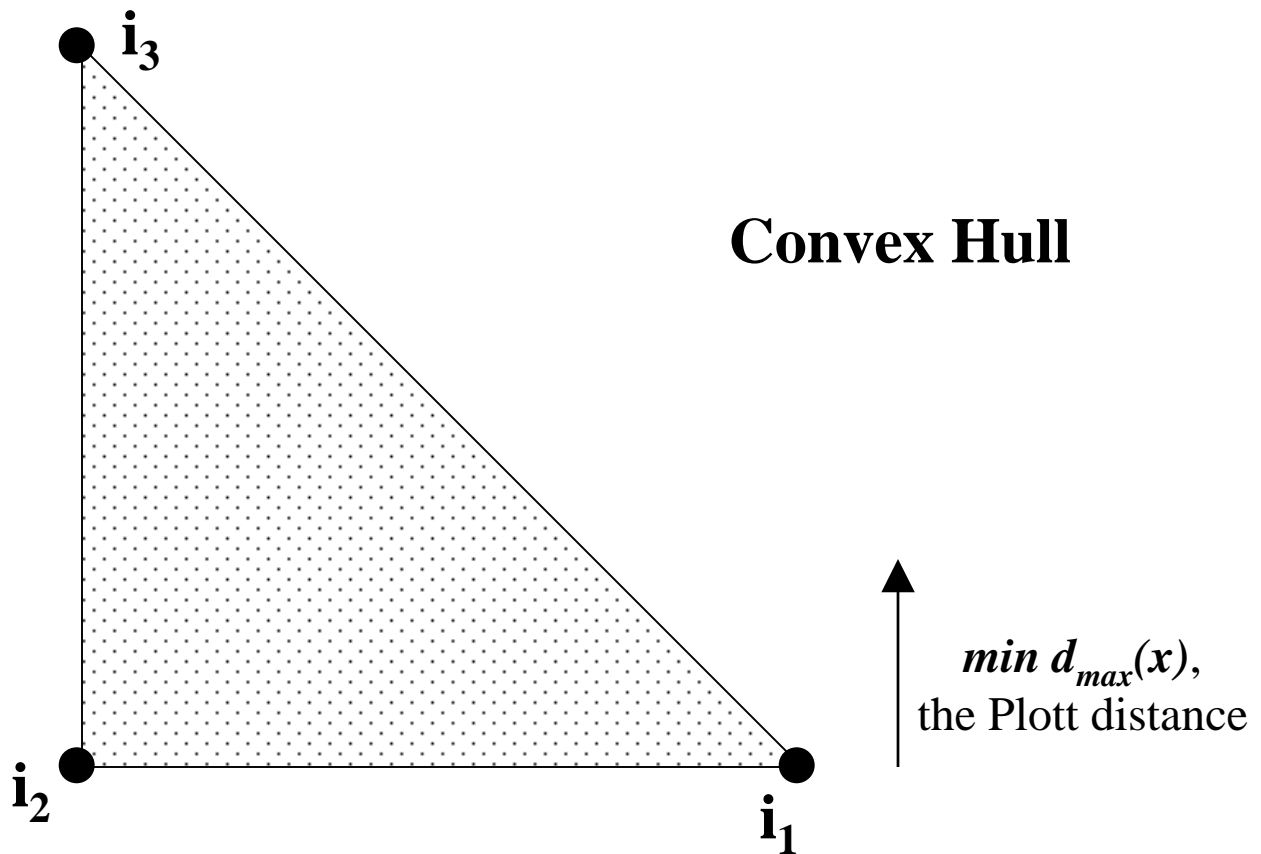
**Figure 4.** Multidimensional example 2



The Plott distance,  $\min d_{\max}(x)$ ,  
as the ideal point  $\mathbf{i}_3$  aligns with  $\mathbf{i}_1, \mathbf{i}_2$ .  
In the limit, the Plott distance is zero  
and we are in the one dimensional case.



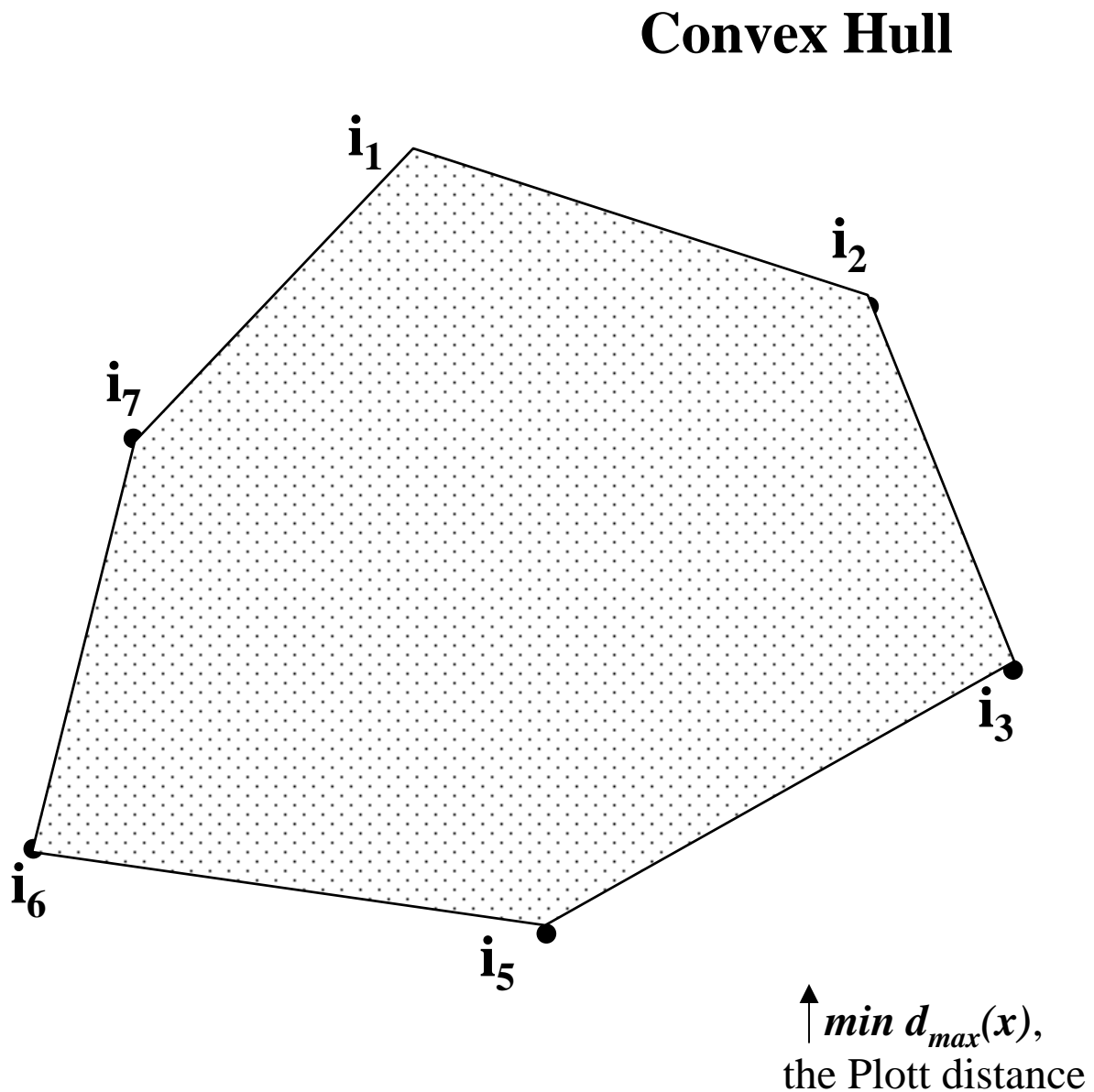
**Figure 5.** The one dimensional example  
as a limit of multidimensional example 1



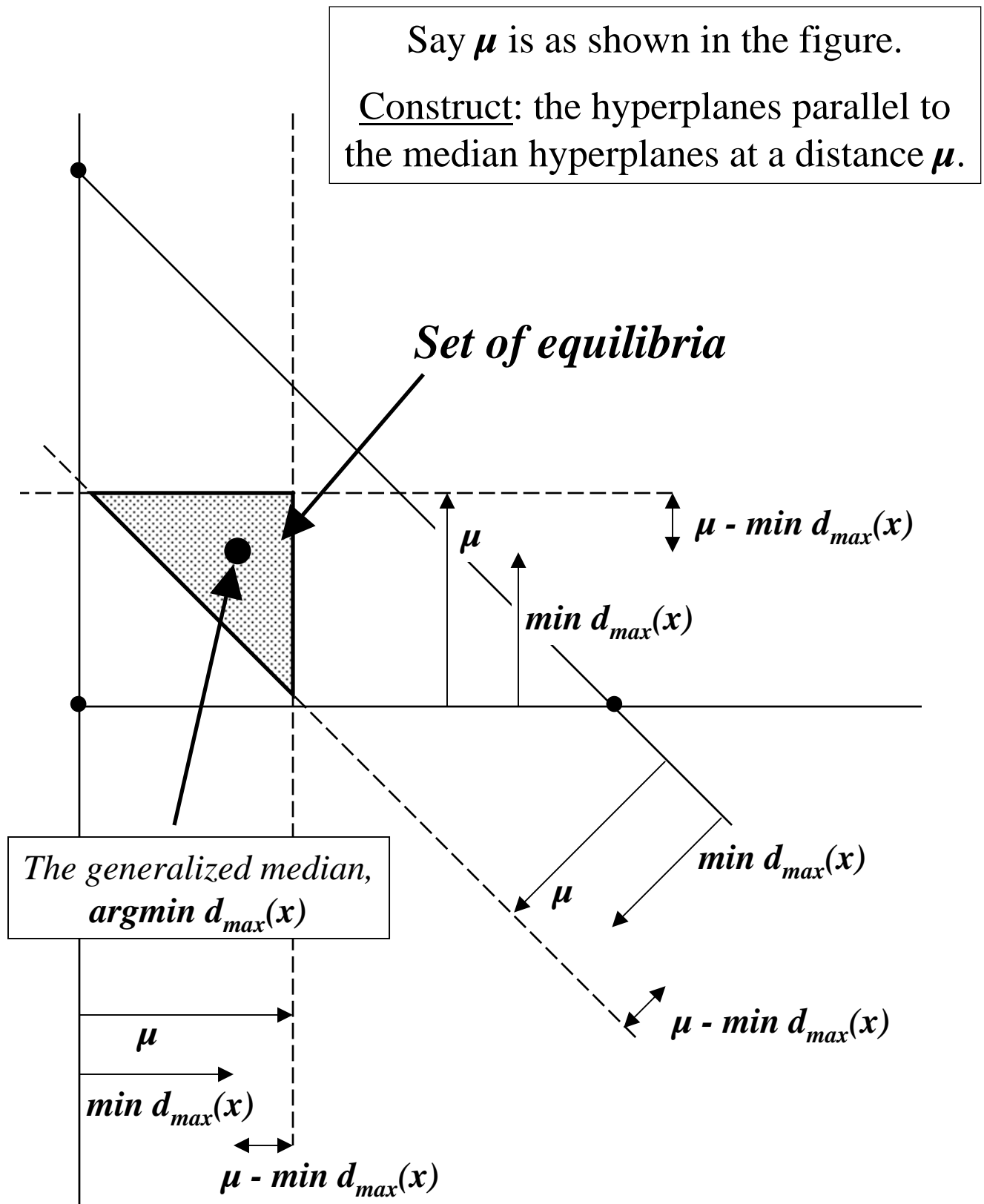
The Plott distance versus the distances between the ideal Points (represented by the convex hull).

**Figure 6.** The distances between ideal points for multidimensional example 1

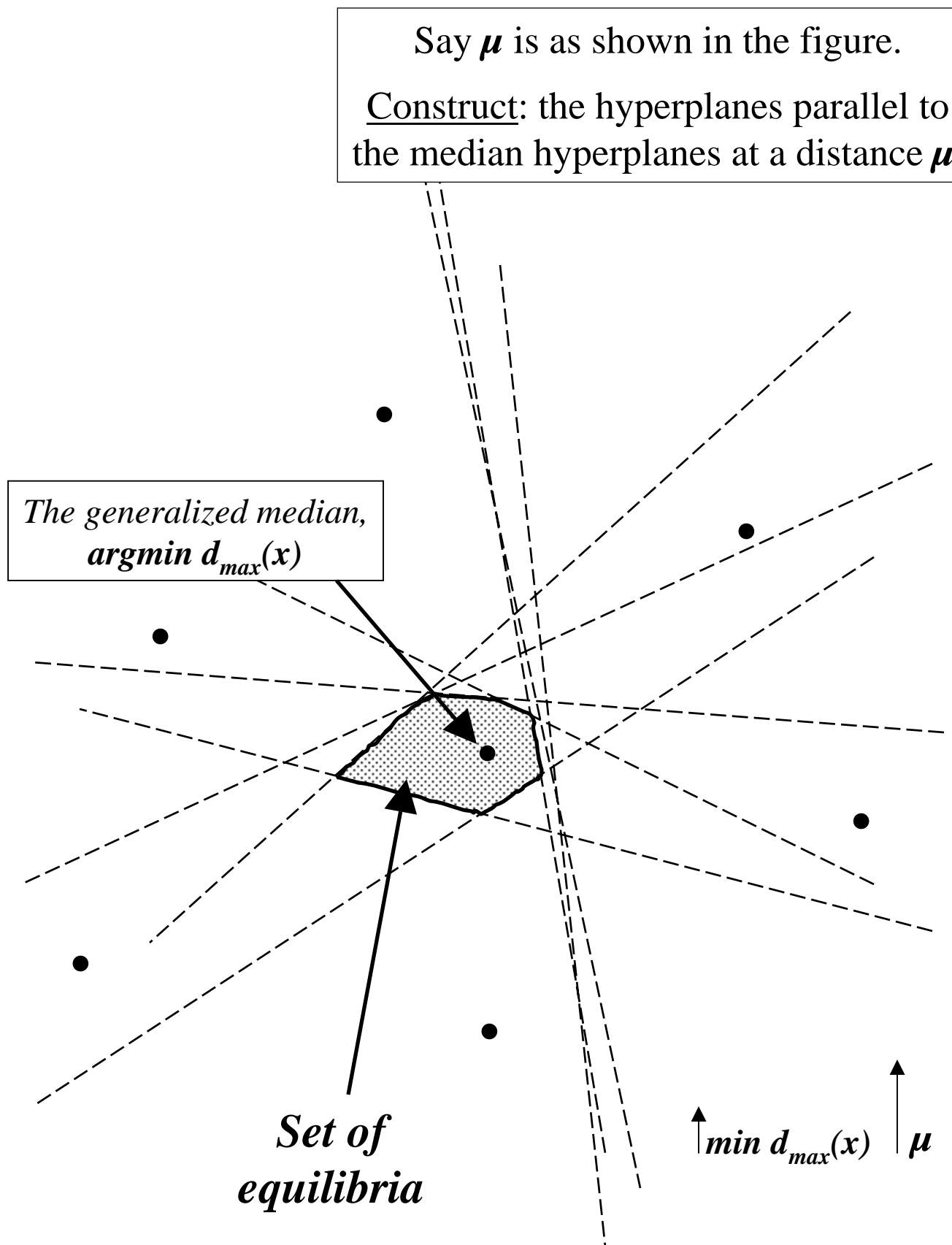
The Plott distance versus the distances between the ideal Points (represented by the convex hull).



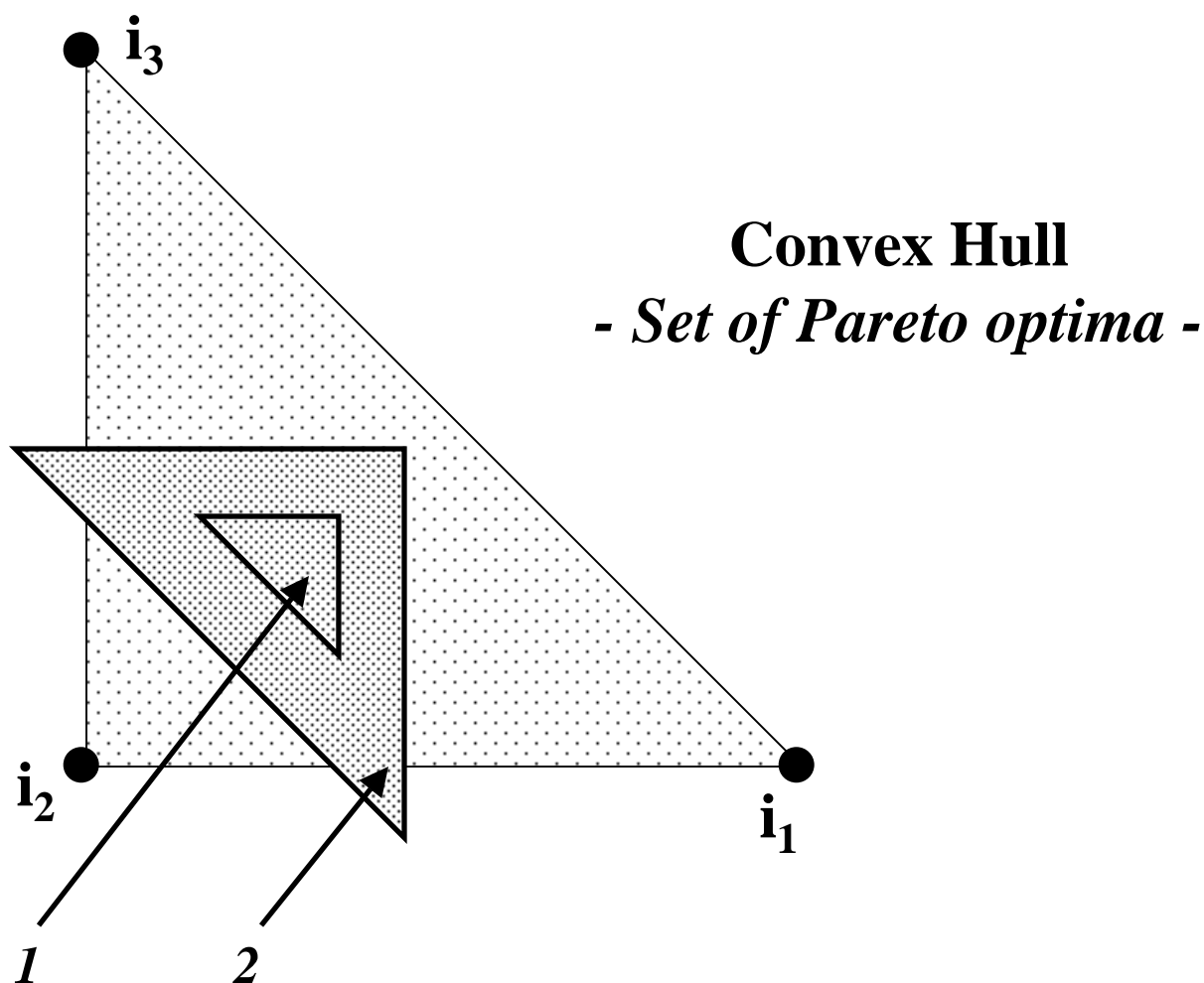
**Figure 7.** The distances between ideal points for multidimensional example 2



**Figure 8.** The set of equilibria, for a given qualitative advantage  $\mu$ , for multidimensional example 1

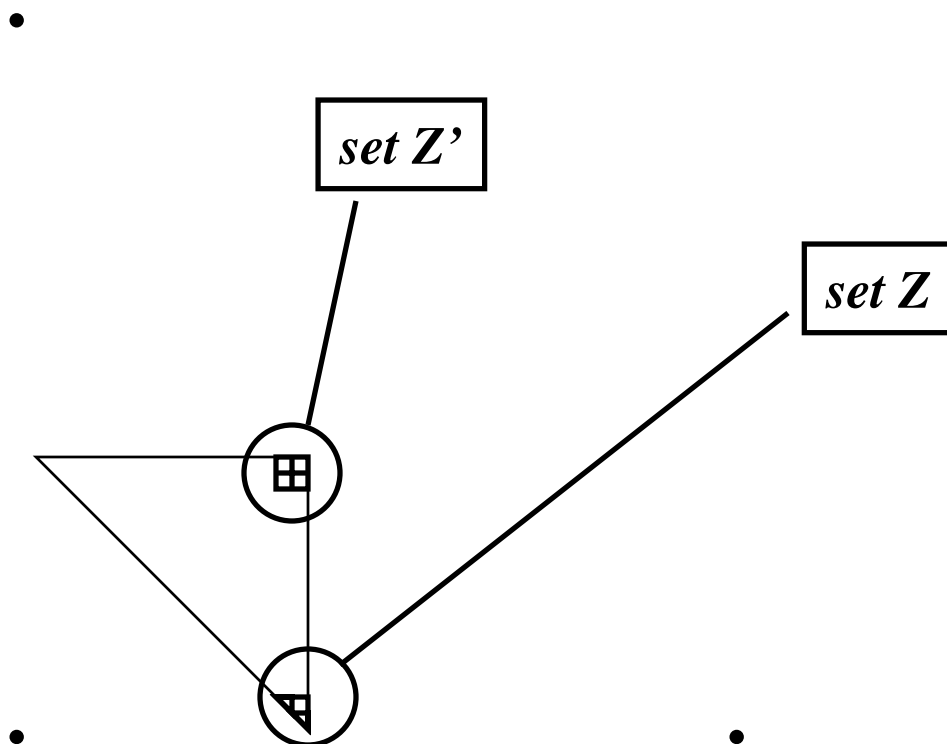


**Figure 9.** The set of equilibria, for a given qualitative advantage  $\mu$ , for multidimensional example 2



There are always equilibria that are Pareto optimal, but not all need be (the **set of equilibria** is shown for two values of  $\mu$  - in case 2 some, but not all, are optimal).

**Figure 10.** The set of Pareto Optima for multidimensional example 1



*In a pair-wise comparison, all points in the set  $Z$  beat all points in the set  $Z'$ , if voters hold the prior belief.*

**Figure 11.** An example of  $Z$  and  $Z'$  for multidimensional example 1

*Say  $s_t > 0$ ,  $R$  is then  
the high-quality candidate,  
and he offers an  $x_{Rt} \in Z'$ .*

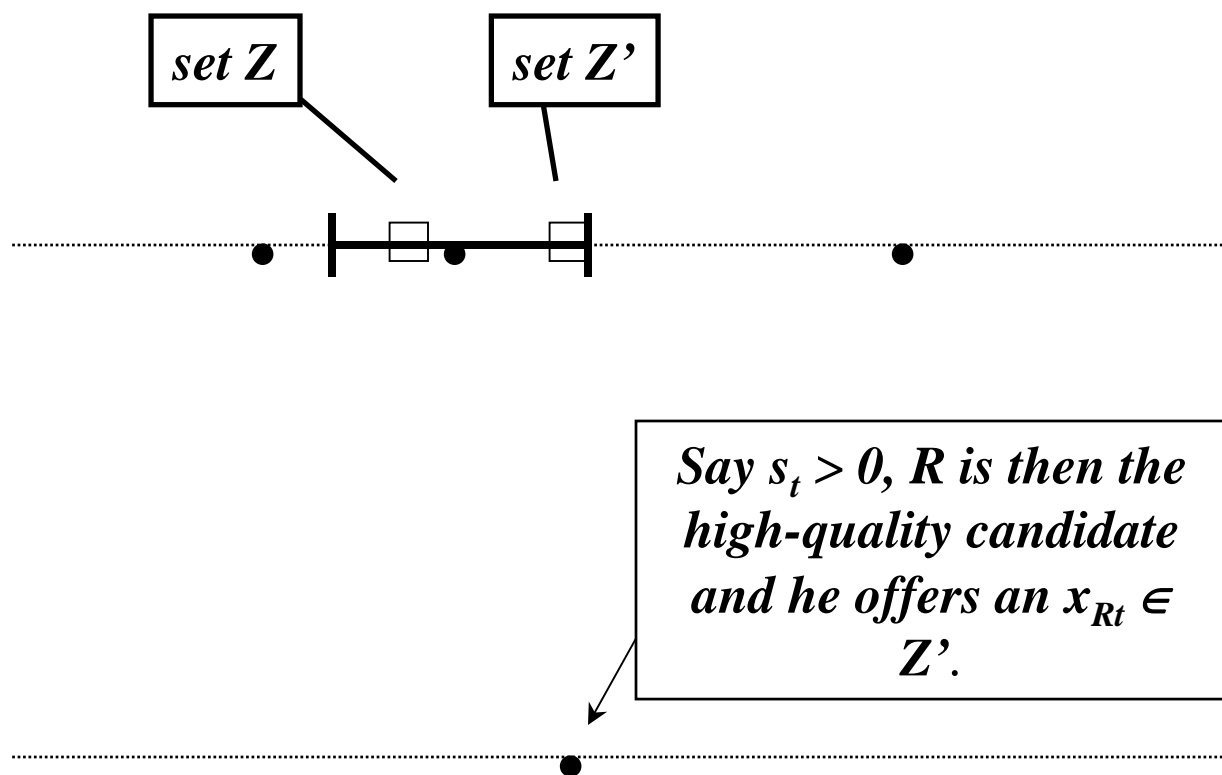
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## *In Equilibrium*

*Voters only observe  
one candidate sitting  
at a point in  $Z$  or  $Z'$ .  
The other is indeterminate.*

**Figure 12.** Observed actions, when there is imperfect information, in multidimensional example 1





Example :  
*In a pair-wise comparison, all points in the set  $Z$  beat all points in the set  $Z'$ , if voters hold the prior belief.*  
In Equilibrium :  
*Voters only observe one candidate sitting at a point in  $Z$  or  $Z'$ , the other is indeterminate.*

**Figure 13.** Figures 11 and 12  
 for the one dimensional example